Minimum Induced Drag of Semi-Elliptic Ground Effect Wing

Hiroshi Mamada*
Aichi University, Aichi, Japan
and
Shigenori Ando†
Nagoya University, Nagoya, Japan

Theme

FOR several years, interest in the Ground Effect Wing (GEW) for future high-speed overwater vehicles has increased. Ando,[1] and Mamada and Ando[2] presented theories on the minimum induced drag of hemicircular front-view GEW. Ashill[3] also published an approximate theory for the ⊓ shaped GEW. Theoretical characteristics of GEW in the limit of vanishing tip gaps have been well established.[1,2] On the other hand, the studies of the influence of GEW front-view profile and other geometrical parameters are rather limited. In the present paper, the minimum induced drag is discussed for a hemi-elliptic front view wing. The main aim is to derive explicit expressions for span efficiency factor, $e(\epsilon,\lambda)$; λ denotes the height ratio of the wing front-view.

Contents

We assume, as in the previous papers,[1,4] that the trailing vortex sheet extends to infinite downstream, retaining the same front-view as that of the wing. Fig. 1 shows a hemi-elliptic front-view wing being in proximity to the ground surface, together with relevant notations. Denote the strength of the trailing vortex sheet per unit length as $\gamma(\Theta)$. Biot-Savart's law gives the normal downwash $w(\Theta)$ on the lifting line as follows

$$w(\theta) = -\frac{1}{4\pi}\int_{\epsilon}^{\pi-\epsilon}\gamma(\theta_1)\Phi(\theta,\theta_1)\frac{\sin\theta_1 d\theta_1}{\cos\theta-\cos\theta_1} \tag{1}$$

The function $\Phi(\Theta,\Theta_1)$ was given in Ref. 4. Equation (1) is regarded as a complicated integral equation for $\gamma(\Theta)$ when $w(\Theta)$ is prescribed. The integration range of Eq. (1) is changed to $0 \le \eta \le \pi$ by introducing a new variable

$$\nu\cos\eta = \cos\theta, \quad \text{with} \quad \nu \equiv \cos\epsilon$$

In addition, to simplify the integrand of Eq. (1) $\tilde{w}(\eta)$ and $\tilde{\gamma}(\eta)$ are introduced for $w(\eta)$ and $\gamma(\eta)$ as follows

$$\left.\begin{aligned}\tilde{w}(\eta) &= w(\eta)(1-\nu^2(1-\lambda^2)\cos^2\eta)^{1/2}\\ \tilde{\gamma}(\eta) &= \gamma(\eta)(1-\nu^2(1-\lambda^2)\cos^2\eta)^{1/2}\end{aligned}\right\} \tag{2}$$

A series expansion for the trailing vortex is assumed

$$\tilde{\gamma}(\eta) \equiv b_0\cot\eta + \sum_{n=1}^{\infty} b_n \sin 2n\eta \tag{3}$$

which corresponds to a symmetric aerodynamic loading having a reasonable singularity at both tips, $\eta = 0$ and π. Equation (3) has been justified for the hemi-circular case.[1,2]

Combining Eqs. (1) to (3), an equation for the unknown coefficients b_0, b_1, ... is obtained. For the sake of simplicity, only two limiting cases of $\epsilon \simeq 0$ and $\pi/2$ are treated; the former case corresponds to a GEW, and the latter demonstrates validity of our analysis for a plane wing out of ground effect. Furthermore $\tilde{w}(\eta)$ is restricted to that of the optimum lift distribution. The modified free vortex strength $\tilde{\gamma}(\eta)$ is obtained from Eq. (4). The spanwise bound vortex distribution $\Gamma(\eta)$ is in turn determined by integrating $\gamma(\eta)$ from zero to η. The span efficiency factor e is defined by

$$e = C_L{}^2/C_D\pi AR \tag{4}$$

which for the optimum case reduces to[5]

$$e = (1/[(R\nu)^2\pi w_{\eta=\pi/2}])\int_{-R\nu}^{+R\nu}\Gamma(\eta)dx \tag{5}$$

where $x = R\nu\cos\eta$. Using the relations obtained above for the case $\nu \sim 1$ ($\epsilon \sim 0$), one obtains the formula shown in Fig. 1, where

$$e_0(\nu \sim 1) = (8/\pi^2)[\ln(1/\sin\epsilon) + 0.88629] \tag{6}$$

The span efficiency factor for a hemi-circular front view GEW is e_0. Figure 1 shows the effect of λ on e. In the case of $\nu \sim 0$ ($\epsilon \simeq \pi/2$),

$$e(\nu \cong 0,\lambda) = [1 + (\nu^2/4) + 0(\nu^4)] + [(1-\lambda^2)/\lambda]^2\nu^2/8 + 0(\nu^4) \tag{7}$$

The first term in [ ] on right hand side is an approximation for e_o when $\nu \cong 0$.

Results and Discussions

In the preceding section, expressions were derived for $e(\lambda)$ in the two limiting cases of $\epsilon \simeq 0$ and $\pi/2$. For each case, the factor $e(\lambda)$ is expressed as the summation of the corresponding $e_0 = e$ ($\lambda = 1$) and the terms containing the height ratio λ, for the each case. It is easily proved that Eq. (7) and the formula in Fig. 1 are accurate not only in the case of $1 \ge \lambda > 0$ but also $\lambda \ge 1$. In fact using the relations $B^2 = [(1-\lambda)/(1+\lambda)]^2 = \{[1-(1/\lambda)]/[1+(1/\lambda)]\}^2$ and $[(1-\lambda^2)/\lambda]^2 = [(1/\lambda)-\lambda]^2$ we may conclude that, at least when $\epsilon \cong 0$ and $\epsilon \cong \pi/2$, the span efficiency factor has the following important property,

$$e(\epsilon,\lambda) = e(\epsilon,1/\lambda) \tag{8}$$

There is no reason to expect that Eq. (8) is not valid for arbitrary ϵ.

The physical meaning of the results is now discussed. The limit $\lambda \to 0$ means the wing approaches the ground throughout its span, and hence may be expected to tend to the ground effect in the ordinary sense for a plane wing. The wing tips play an insignificant role as end-plates. When $\lambda \to \infty$, the central part of the wing is elevated out

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*Associate Professor of Department of Physics, Aichi University of Education.

†Professor of Department of Aeronautical Engineering. Nagoya University.

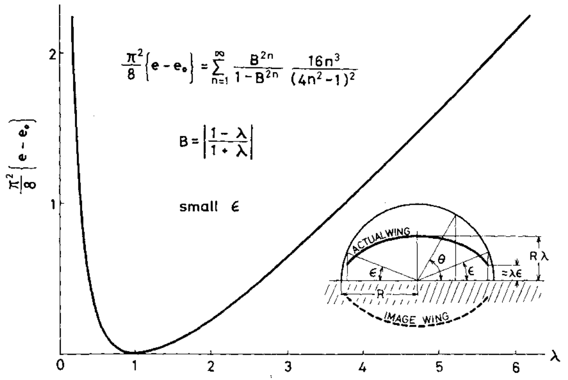

Fig. 1 Span efficiency factor is symmetrical in exchanging λ for 1/λ, and minimum at λ = 1.

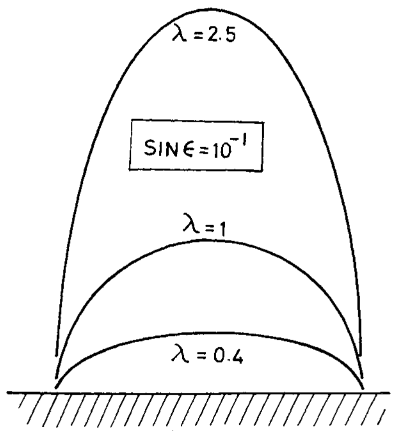

Fig. 2 Two wings with λ = 0.4 and λ = 1/0.4 have a same span efficiency factor *e*.

of ground effect. Hence, it might be thought that the ground effect in the ordinary sense decreases. Nevertheless, an increase in the span efficiency factor was obtained not only in the case of diminishing λ but also enlarging λ. This implies the the wing tips become more effective as end-plates in the case of $\lambda \to \infty$.

Equation (8) implies that these two effects, the ground effect in the conventional sense and the end-plate effect, are equivalent for identical ϵ in the particular case when the front view of the wing is hemi-elliptic. It should be noted that the wing tip gap width does not remain constant. See Fig. 2, where two wings with $\lambda = 0.4$ and $\lambda = 2.5$ have equal span efficiency factors, in spite of the difference in the gap widths. It should be noted that the wing with the *hemi-circular* front view has the least span efficiency factor of the three GEW in Fig. 2 all with a common gap parameter ϵ.

A few result obtained from the formula in Fig. 1 are now discussed for the particularly interesting case of $\nu \sim 1$ ($\epsilon \sim 0$). First, consider the limit when the height ratio λ tends to zero. Since $\lambda \to 0$ corresponds to $B^2 \to 1$, it is found that the main contribution to e originates from the summation over n rather than e_0 in Eq. (6). Using the approximate equation

$$\frac{B^{2n}}{1-B^{2n}} = \frac{B^2}{1-B^2}\,\frac{B^{2n-2}}{1+B^2+\ldots+B^{2n-2}} \lesssim \frac{B^2}{1-B^2}\,\frac{1}{n}$$

the summation shown in Fig. 1 becomes

$$\sum_{n=1}^{\infty}\frac{B^{2n}}{1-B^{2n}}\,\frac{16n^3}{(4n^2-1)^2} \cong \frac{B^2}{1-B^2}\sum_{n=1}^{\infty}\frac{16n^2}{(4n^2-1)^2} = \frac{(1-\lambda^2)^2}{4\lambda}\cdot\frac{\pi^2}{4} \qquad (9)$$

Then, on the limit $\lambda \to 0$, the span efficiency factor is mainly given by

$$e(\lambda \to 0) \to \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{B^{2n}}{1-B^{2n}}\,\frac{16n^3}{(4n^2-1)^2} \to \frac{1}{2\lambda} \qquad (10)$$

According to P.A. Ashill, the ground effect of a plane wing is given exactly as

$$e \to (2/3\pi)(b/2h)\ (h \to 0) \qquad (11)$$

where b is the span and h denotes the height of the wing above the ground surface. Comparing these two cases, we recognize that $b/2h$ corresponds to $1/\lambda$ since there exist the correspondences of $h \leftrightarrow R\lambda$ and $b \leftrightarrow 2R\nu \sim 2R$. Thus the result

$$e \propto [\text{SPAN}]/[\text{WING HEIGHT}] \qquad (12)$$

is confirmed not only in Eq. (11) but also in Eq. (10). Also, it is anticipated that the hemi-elliptical wing shows somewhat stronger ground effect than a plane wing, because the wing tips are closer to the ground than the plane wing. Since $1/2 > 2/3\pi$ our result is very plausible in fact. (In the limit $\lambda \to \infty$, e is proportional to λ; the physical significance of this limit is not so obvious.)

When $\lambda \sim 1$, namely the front view is nearly hemi-circle, the approximated summation may be as,

$$\sum_{n=1}^{\infty}\frac{B^{2n}}{1-B^{2n}}\,\frac{16n^3}{(4n^2-1)^2} \cong \sum_{n=1}^{\infty} B^{2n}/n = \ln[1/(1-B^2)] = \ln(1/\lambda) + 2\ln[(\lambda+1)/2] \qquad (13)$$

Adding the first term of the above to the first term of Eq. (6), yields the result,

$$\ln[1/(\lambda \sin\epsilon)] \cong \ln([\text{SPAN}]/2[\text{GAP WIDTH}]) \qquad (14)$$

which isolates the effect of the gap width. The other contribution (second term of Eq. 13) comes from the fact that the height of the wing varies as a whole. It is a significant result that, whenever ϵ is sufficiently small and λ does not deviate greatly from $\lambda = 1$, the main contribution to e is due to an expression such as Eq. (14). As a conclusion, when only the wing tips are in close proximity to the ground, the limiting behavior of the span efficiency factor is given by

$$e \propto \ln([\text{SPAN}]/[\text{GAP WIDTH}]) \qquad (15)$$

This agrees with the results obtained by Ashill[4] for the ⊓-shaped wing.

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